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# Univalence and completeness of Segal objects

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# Outline

# Type theoretic model categories

## Definition (ad hoc)

A *type theoretic model category* $\mathbb{M}$ is a model category such that its associated category $\mathbb{C} := \mathbb{M}^f$ of fibrant objects is a type theoretic fibration category.

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The Quillen model structure $(\mathbf{S}, \mathrm{Kan})$.

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## Recall

1. Complete Segal spaces are Reedy fibrant simplicial objects in $(\mathbf{S}, \mathrm{Kan})$ satisfying *the Segal conditions* and *the completeness condition*.

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## Example

The Quillen model structure $(\mathbf{S}, \mathrm{Kan})$.

## Recall

2. There is a model structure $(s\mathbf{S}, \mathrm{CS})$ whose fibrant objects are the complete Segal spaces.
   $\rightsquigarrow$ Classical model for $(\infty, 1)$-category theory.

Fix a type theoretic model category $\mathbb{M}$ with associated category $\mathbb{C}$ of fibrant objects.

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& 1 \arrow[dr, bend left] & \\
0 \arrow[ur, bend left] \arrow[rr, bend right] & & 2
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$$\begin{tikzcd}
 & 1 \arrow[dr] \arrow[rr, bend left] & & 3 \\
0 \arrow[ur] \arrow[rr, bend right] \arrow[urrr, dashed] & & 2 \arrow[ur] &
\end{tikzcd}$$

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Definition
The *n-th Segal map* associated to a simplicial object $X$ in $\mathbb{M}$ is the map

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The *n-th Segal map* associated to a simplicial object $X$ in $\mathbb{M}$ is the map

$$\xi_n \colon X_n \to (X_{1/X_0})^n_S.$$

Definition

Let $X \in s\mathbb{C}$ be a simplicial object in $\mathbb{C}$.

1. $X$ is *sufficiently fibrant* if both the 2-Segal map

$$\xi_2 \colon X_2 \to X_1 \times_{X_0} X_1$$

   and the boundary map

$$(d_1, d_0) \colon X_1 \to X_0 \times X_0$$

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2. Let $X$ be sufficiently fibrant. We say that $X$ is a *Segal object* (*strict Segal object*) if the associated Segal maps

$$\xi_n \colon X_n \to (X_{1/X_0})^n_S$$

   are homotopy equivalences (isomorphisms) in $\mathbb{C}$.

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for

$$\mathrm{Linv}(x,y,f) :\equiv \sum_{g:X_1(y,x)} \sum_{\sigma:X_2(f,g)} d_1\sigma =_{X_1(x,x)} s_0 x,$$

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- There is a nerve construction

$$N\colon \mathrm{ICat}(\mathbb{C}) \to s\mathbb{C}$$

  whose image consists exactly of the objects in $s\mathbb{C}$ whose Segal objects are isomorphisms.

$\rightsquigarrow$ $N\mathrm{Fun}(p)$ is a strict Segal object.

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$\rightsquigarrow$ $N\mathrm{Fun}(p)$ is a strict Segal object.

Proposition

*Let $p\colon E \twoheadrightarrow B$ be a fibration in $\mathbb{C}$. Then $p$ is a univalent fibration in $\mathbb{C}$ if and only if the Segal object $N\mathrm{Fun}(p)$ is univalent.*

# Rezk Completeness

Let $X \in s\mathbb{C}$ be a simplicial object in $\mathbb{C}$. Recall the Kan extension

$$\begin{tikzcd} \Delta^{op} \arrow[r, "X"] \arrow[d, "y^{op}"'] & \mathbb{M} \\ (\mathbf{S})^{op} \arrow[ur, dashed, "\_\ \backslash X := \mathrm{Ran}_{y^{op}} X"'] & \end{tikzcd}$$

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Definition
A Reedy fibrant Segal object $X$ is *complete* if the functor

$$\_ \setminus X \colon (\mathbf{S}, \mathrm{QCat})^{op} \to \mathbb{M}$$

is a right Quillen functor.

A map $\mathcal{C} \to \mathcal{D}$ between quasi-categories is a quasi-fibration if and only if it has the right lifting property against

1. all inner horn inclusions $\{h_i^n \colon \Lambda_i^n \to \Delta^n\}$, and
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$$J = N\left(\begin{tikzcd} 0 \arrow[r, bend left] \arrow[r, phantom, "\cong"] & 1 \arrow[l, bend left] \end{tikzcd}\right)$$

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$$
K = \begin{tikzcd}
 & 1 \arrow[rr, bend left, "s_0 1"] \arrow[dr, bend left, "f"] & & 1 \\
0 \arrow[ur, bend left, "g"] \arrow[rr, bend right, "s_0 0"'] & & 0 \arrow[ur, bend right, "h"'] &
\end{tikzcd}
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\mathrm{Equiv}(X) \arrow[dr] & X_0 \times X_0 & K \setminus X \arrow[dl] \\
\end{tikzcd}$$

# Comparison of univalence and completeness

For every Segal object $X \in s\mathbb{C}$, we have

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Theorem

*Let $X$ be a Segal object in $\mathbb{C}$. Then $X$ is univalent if and only if its Reedy fibrant replacement $\mathbb{R}X$ is complete.*

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Corollary

*Let $X$ be a fibration in $\mathbb{C}$. Then $p$ is a univalent fibration if and only if the Segal object $\mathbb{R}N(\mathrm{Fun}(p))$ is complete.*

# Univalent and Rezk completion

# Univalent completion

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Let $\pi \colon \tilde{U} \twoheadrightarrow U$ be a univalent fibration.

Definition

We say that $p \colon E \twoheadrightarrow B$ is *small* if it arises as the homotopy pullback of $\pi$ along some map $B \to U$.

Definition

Let $p\colon E \twoheadrightarrow B$ be a small fibration in $\mathbb{C}$. We say that a homotopy cartesian square

$$\begin{tikzcd} E \arrow[r] \arrow[d, two heads, "p"'] & u(E) \arrow[d, two heads, "u(p)"] \\ B \arrow[r, "\iota"'] & u(B) \end{tikzcd}$$

is a *univalent completion* of $p$ if the fibration $u(p) \in \mathbb{C}$ is small and univalent, and the map $\iota\colon B \to u(B)$ is a $(-1)$-connected cofibration.

## Proposition

*For every fibration $p \colon E \twoheadrightarrow B$ in $\mathbb{C}$ there is a univalent completion*

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## Proof.

$$\begin{tikzcd} E \arrow[r] \arrow[d, two heads, "p"'] \arrow[dr, phantom, "\lrcorner", very near start] & \tilde{U} \arrow[d, two heads, "\pi"] \\ B \arrow[r, "b"'] & U \end{tikzcd}$$

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□

# Rezk completion

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Given a map $f \colon X \to Y$ of Segal objects in $\mathbb{C}$, consider

$$
\begin{tikzcd}
\mathrm{Eq} f \arrow[r] \arrow[d, two heads] \arrow[dr, phantom, "\lrcorner", very near start] & \mathrm{Equiv}\, Y \arrow[d, two heads] \\
f \downarrow Y \arrow[r] \arrow[d, two heads] \arrow[dr, phantom, "\lrcorner", very near start] & Y_1 \arrow[d, two heads, "(s,t)"] \\
X_0 \times Y_0 \arrow[r, "f_0 \times 1"'] & Y_0 \times Y_0
\end{tikzcd}
$$

Definition

Let $f\colon X \to Y$ be a map between Segal objects in $\mathbb{M}$. We say that

1. $f$ is *fully faithful* if the natural map $X_1 \to (f_0 \times f_0)^* Y_1$ over $X_0 \times X_0$ is a weak equivalence.
2. $f$ is *essentially surjective* if the fibration $(\mathrm{Eq} f)_{-1} \twoheadrightarrow Y_0$ is acyclic.
3. $f$ is a *DK-equivalence* if it is fully faithful and essentially surjective.

Theorem

*For every fibration $p\colon E \twoheadrightarrow B$, the univalent completion*

$$\begin{tikzcd} E \arrow[r] \arrow[d, two heads, "p"'] & u(E) \arrow[d, two heads, "u(p)"] \\ B \arrow[r, "\iota"'] & u(B) \end{tikzcd}$$

*induces a DK-equivalence*

$$\mathbb{R}N(\iota)\colon \mathbb{R}N(p) \to \mathbb{R}N(u(p))$$

*from the Segal object $\mathbb{R}N(p)$ to the complete Segal object $\mathbb{R}N(u(p))$ in $\mathbb{C}$.*

# Outlook

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- Deduce existence of Rezk completion from univalent completion and vice versa, depending on the context.

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- Deduce existence of Rezk completion from univalent completion and vice versa, depending on the context.
- Coming from simplicial homotopy theory, this suggests that univalent fibrations might be the fibrant objects in some fibration category?

# Thank you!


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