Cubical Exact Equality and Categorical Gluing

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Presenting zero-dimensional structures

Equational axioms of zero-dimensional structures can be presented in many ways:

- "tacit" equality / equality reflection: ETT, HTS, ZF, Nuprl, etc. [ML79; Con+86; Voe13; Bau+16; AFH18; Ang+18]
- "coercive" or transportative equality: ITT, HoTT, OTT, etc.
  - weakly coherent: hSets, i.e. types with UIP [Uni13; Str94]
  - strictly coherent: Bishop sets, i.e. types with "judgmental UIP" [Coq17; Mou+15; SAG19; AM06]

In **informal mathematics**, tacit equality is often preferred (no fuss, no muss). In **proof assistants**, coercive equality usually works better (really!), but presents its own difficulties.

Strictly coherent presentations favored for efficiency, ease of use.

Desirable properties of presentations

Type theorists and implementers of proof assistants have found these ~~properties~~ structures to be essential, in order of increasing strength:

- **Canonicity:** global elements of base type are equal to constants ($\Longrightarrow$ operational/computational semantics)
- **Normalization:** canonical representatives of judgmental/definitional equivalence classes for generalized elements ($\Longrightarrow$ decidability of the word problem)
- **Type checking:** elaboration of concrete syntax to abstract syntax (preferred decidable)

Normalization and type checking incompatible with tacit equality (but canonicity is easier).

Coercive equality and extensionality

In ITT, coercive equality is governed by Martin-Löf's *identity type* $\mathbf{Id}_A(M, N)$, generated by $\mathbf{refl}_M : \mathbf{Id}_A(M, M)$.

Further extensionality principles (e.g. for functions, sets) are *not* derivable, but can be consistently added as axioms (disrupting canonicity).

Inspired by semantic models (setoids, groupoids, cubical sets), newer type theories combine coercive equality with extensionality principles while preserving canonicity:

- Cubical Type Theory [CCHM17; AFH17; ABCFHL]: infinite-dimensional type theory
- OTT [AM06; AMS07]: set-level type theory (setoid-style)
- **XTT [SAG19]: set-level type theory (cubical-style)**, a strictly truncated version of Cartesian Cubical Type Theory [ABCFHL] for which we have proved canonicity

Equality types from an interval (syntactically)

$$\frac{\Gamma\ \mathit{ctx}}{\Gamma, i : \mathbb{I}\ \mathit{ctx}} \qquad\qquad \frac{\Gamma\ \mathit{ctx} \qquad \Gamma \vdash r : \mathbb{I} \qquad \Gamma \vdash r' : \mathbb{I}}{\Gamma, r = r'\ \mathit{ctx}}$$

Equality types from an interval (syntactically)

FORMATION

$$\frac{\Gamma, i : \mathbb{I} \vdash A[i] \ \mathit{type} \qquad \overrightarrow{\Gamma \vdash N_\varepsilon : A[\varepsilon]} \quad (\varepsilon \in \{0,1\})}{\Gamma \vdash \mathbf{Eq}_{i.A[i]}(N_0, N_1) \ \mathit{type}}$$

INTRODUCTION

$$\frac{\Gamma, i : \mathbb{I} \vdash M : A \qquad \overrightarrow{\Gamma, i : \mathbb{I}, i = \varepsilon \vdash M \equiv N_\varepsilon : A[i]} \quad (\varepsilon \in \{0,1\})}{\Gamma \vdash \lambda i.M : \mathbf{Eq}_{i.A[i]}(N_0, N_1)}$$

ELIMINATION

$$\frac{\Gamma \vdash M : \mathbf{Eq}_{i.A[i]}(N_0, N_1) \qquad \Gamma \vdash r : \mathbb{I}}{\Gamma \vdash M(r) : A[r] \qquad \Gamma \vdash M(\varepsilon) \equiv N_\varepsilon : A[\varepsilon] \quad (\varepsilon \in \{0,1\})}$$

COMPUTATION

$$\frac{\Gamma, i : \mathbb{I} \vdash M[i] : A[i] \qquad \Gamma \vdash r : \mathbb{I}}{\Gamma \vdash (\lambda i.M[i])(r) \equiv M[r] : A[r]}$$

UNICITY

$$\frac{\Gamma \vdash M : \mathbf{Eq}_{i.A[i]}(N_0, N_1)}{\Gamma \vdash M \equiv \lambda i.M(i) : \mathbf{Eq}_{i.A[i]}(N_0, N_1)}$$

Equality types from an interval (semantically)

Let $\mathbb{C}$ be our category of contexts; let $\mathbb{I} : \mathbf{Pr}(\mathbb{C})$ be a representable interval object, and let $\widetilde{\mathscr{U}} \xrightarrow{\varpi} \mathscr{U}$ be a natural model over $\mathbb{C}$ in the sense of Awodey [Awo18]. Then, the equality type connective is specified by the following pullback square:

$$
\begin{tikzcd}
\widetilde{\mathscr{U}}_{\mathrm{Eq}} \arrow[r, dotted, "\mathbf{lam}"] \arrow[d, "\varpi_{\mathrm{Eq}}"'] \arrow[dr, phantom, "\lrcorner", very near start] & \widetilde{\mathscr{U}} \arrow[d, "\varpi"] \\
\mathscr{U}_{\mathrm{Eq}} \arrow[r, dotted, "\mathbf{Eq}"'] & \mathscr{U}
\end{tikzcd}
$$

$$
\begin{aligned}
\widetilde{\mathscr{U}}_{\mathrm{Eq}} &= \widetilde{\mathscr{U}}^{\mathbb{I}} \\
\mathscr{U}_{\mathrm{Eq}} &= \textstyle\sum_{A:\mathscr{U}^{\mathbb{I}}} \prod_{i:\mathbb{I}} \partial i \to Ai \\
\varpi_{\mathrm{Eq}} &= \lambda M.(\varpi \circ M, \lambda i \alpha. Mi)
\end{aligned}
$$

Strict UIP via boundary separation (syntactically)

XTT is about sets, i.e. types with UIP; we choose to impose a strict version of UIP in which any two elements of $\mathbf{Eq}_A(N_0, N_1)$ are judgmentally equal. We state this independently of the equality connective:

$\Gamma \vdash r : \mathbb{I}$

TYPE BOUNDARY SEPARATION

$$\frac{\overrightarrow{\Gamma, r = \varepsilon \vdash A \equiv B\ \mathit{type}} \quad (\varepsilon \in \{0,1\})}{\Gamma \vdash A \equiv B\ \mathit{type}}$$

TERM BOUNDARY SEPARATION

$$\frac{\overrightarrow{\Gamma, r = \varepsilon \vdash M \equiv N : A} \quad (\varepsilon \in \{0,1\})}{\Gamma \vdash M \equiv N : A}$$

Derivable: $\Gamma \vdash P \equiv Q : \mathbf{Eq}_{i.A[i]}(N_0, N_1)$ given $\Gamma \vdash P, Q : \mathbf{Eq}_{i.A[i]}(N_0, N_1)$.

Strict UIP via boundary separation (semantically)

Semantically, strict UIP arises by requiring that the presheaves of types and elements be *separated* relative to a certain coverage on $\mathbb{C}$:

$$\mathbf{K}_\partial(\Gamma) \ni \left\{\Gamma.\partial r \longrightarrow \Gamma\right\} \text{ for each } r \in \mathbb{I}(\Gamma)$$

If $\mathscr{U}, \widetilde{\mathscr{U}} : \mathbf{Pr}(\mathbb{C})$ are $\mathbf{K}_\partial$-separated, then each $\varpi[\mathbf{Eq}_A(M,N)]$ is strictly subsingleton.

Coquand considered a version of this condition for universes of *Bishop sets* [Coq17].

Generalized composition (syntactically)

We can coerce between any two points on a line, optionally attaching two faces along the boundary of any dimension $s : \mathbb{I}$.

$$\Gamma, i : \mathbb{I} \vdash A[i]\ \mathit{type} \qquad \Gamma \vdash r, r' : \mathbb{I} \qquad \Gamma \vdash M : A[r]$$

COERCION

$$\frac{}{\Gamma \vdash i.A[i] \downarrow^{r}_{r'} M : A[r'] \qquad \Gamma, r = r' \vdash i.A[i] \downarrow^{r}_{r'} M \equiv M : A[r']}$$

COMPOSITION

$$\frac{\Gamma \vdash s : \mathbb{I} \qquad \overrightarrow{\Gamma, s = \varepsilon, i : \mathbb{I} \vdash N_\varepsilon[i] : A[i]} \qquad \overrightarrow{\Gamma, s = \varepsilon \vdash N_\varepsilon[r] \equiv M : A[r]}}{\begin{array}{c} \Gamma \vdash i.A[i] \downarrow^{r}_{r'} [M \mid \overrightarrow{s = \varepsilon \to i.N_\varepsilon[i]}] : A[r'] \\ \Gamma, r = r' \vdash i.A[i] \downarrow^{r}_{r'} [M \mid \overrightarrow{s = \varepsilon \to i.N_\varepsilon[i]}] \equiv M : A[r'] \\ \Gamma, s = \varepsilon \vdash i.A[i] \downarrow^{r}_{r'} [M \mid \overrightarrow{s = \varepsilon \to i.N_\varepsilon[i]}] \equiv N_\varepsilon[r'] : A[r'] \end{array}}$$

Generalized composition (semantically)

"Tube shapes" $\varphi : \mathbb{F}$ are generated by $\bot$ and $\partial(s)$ for any $s : \mathbb{I}$. Internally to the logical framework of $\mathbf{Pr}(\mathbb{C})$ (see [OP16]), we have the following orthogonality structure for each display map $\varpi[A] \xrightarrow{\mathbf{p}_A} よ\Gamma$, given a dimension $r : \mathbb{I}$ and a tube shape $\varphi : \mathbb{F}$:

$$
\begin{tikzcd}
\sum_{i:\mathbb{I}}[(i = r) \vee \varphi] \arrow[r, "S"] \arrow[d, tail] & \varpi[A] \arrow[d, "\mathbf{p}_A"] \\
\mathbb{I} \arrow[r, "\gamma"'] \arrow[ur, dotted, "A\gamma \downarrow^{r}_{\bullet} S"] & よ\Gamma
\end{tikzcd}
$$

(Same filling problems as Angiuli, Harper, and Wilson [AHW16], except we impose *regularity*)

Hierarchies of universes of sets

In higher-dimensional semantics, a (standard) universe of sets is not a set. But this higher-dimensional data can be controlled by equipping the standard universes with a structure, for instance:

- *well-ordering structure*, as in the interpretation of cumulative set theories into type theory [Acz78; Uni13]
- ***inductive-recursive structure***, as in Martin-Löf's earliest accounts of hierarchies of set-universes [ML84]

For subtle reasons concerning the word problem for XTT, we equip the standard universes with IR codes.

Objective metatheory of XTT

XTT is *essentially algebraic*, and therefore obtains a category of algebras with an initial object (the syntactic model) [Car78; PV07; Awo18; Uem19].

Follows from general considerations, and does not require taking a position on the so-called “initiality conjecture”.

XTT’s algebraic model theory enables a presentation-independent proof of *canonicity*.

Canonicity

Canonicity for type theories, analogous to the disjunction property for logics, is a form of "internal constructivity" (computation).

Theorem (Canonicity)
If $\cdot \vdash M : \mathbf{bool}$*, then either* $M \equiv \mathbf{true}$ *or* $M \equiv \mathbf{false}$*.*

Canonicity (and normalization) for algebraic languages is always obtained through *Artin gluing* along a suitable relativization of the global sections functor [LS86; Cro93; AHS95; Str98; Alt+01; Fio02; Shu15; SS18; Coq19; KHS19].

A nerve from XTT into cubical sets

Let $\square$ be the category of cubes, and let $\mathbb{C}$ be the syntactic category of XTT. Because the interval $\mathbb{I}$ is representable, we obtain an embedding $\square \xrightarrow{i} \mathbb{C}$, inducing a *nerve* $\mathbf{N}$ from $\mathbb{C}$ into cubical sets:

$$
\begin{tikzcd}
\mathbb{C} \arrow[d, hook, "\text{よ}"'] \arrow[dr, dotted, "\boldsymbol{N}"] & \\
\mathbf{Pr}(\mathbb{C}) \arrow[r, "i^*"'] & \mathbf{Pr}(\square)
\end{tikzcd}
$$

By *gluing* along this nerve, independently proposed by Awodey, we obtain a *cubical* notion of computability in the sense of Tait and Martin-Löf [Tai67; ML75a]. This is classical global-sections gluing from the internal point of view (see for instance [CHS19]).

Artin gluing for XTT

Using the comma construction, we obtain a category of *cubical logical families* over $\mathbb{C}$:

$$\mathbb{G} = \mathbf{id}_{\mathbf{Pr}(\square)} \downarrow \mathbf{N}$$

An alternative presentation of the gluing category is obtained by pulling back the fundamental fibration along the nerve functor:

$$\begin{tikzcd}
\mathbb{G} \arrow[r, dotted] \arrow[d, dotted, "\partial_1[\mathbf{N}]"'] \arrow[dr, phantom, "\lrcorner", very near start] & {[\mathbb{2}, \mathbf{Pr}(\square)]} \arrow[d, "\partial_1"] \\
\mathbb{C} \arrow[r, "\mathbf{N}"'] & \mathbf{Pr}(\square)
\end{tikzcd}$$

Artin gluing for XTT

Theorem

$\mathbb{G}$ *carries an XTT-algebra structure, and furthermore* $\mathbb{G} \xrightarrow{\partial_1[\mathbf{N}]} \mathbb{C}$ *is a homomorphism of XTT-algebras.*

Sketch.

Obviously $\mathbf{N}$ is a pseudomorphism of natural models in the sense of Kaposi, Huber, and Sattler [KHS19], since $\mathbb{C} \xhookrightarrow{よ} \mathbf{Pr}(\mathbb{C})$ is one, and change of base $\mathbf{Pr}(\mathbb{C}) \xrightarrow{i^*} \mathbf{Pr}(\square)$ is left exact. Therefore, the $(\Pi, \Sigma, \mathbf{bool})$ structure on $\mathbf{Pr}(\square)$ lifts to $\mathbb{G}$; one then checks equality types, IR universes, and composition structures. $\square$

Canonicity from the gluing model

The boolean object $\widetilde{\mathbf{bool}} : \mathbb{G}$ was given by the following family in $\mathbf{Pr}(\square)$:

$$
\begin{tikzcd}
& \mathbf{1} = \mathbf{N}(\mathbf{1}) \arrow[d] \arrow[ddr, bend left, "\mathbf{N}(\mathbf{true})"] & \\
\mathbf{1} = \mathbf{N}(\mathbf{1}) \arrow[r] \arrow[drr, bend right, "\mathbf{N}(\mathbf{false})"'] & \mathbf{1} + \mathbf{1} \arrow[dr, dotted, red, "\widetilde{\mathbf{bool}}"] & \\
& & \mathbf{N}(\mathbf{bool})
\end{tikzcd}
$$

A computable boolean is equipped with a proof that it is one of the two constants!

Canonicity from the gluing model

Theorem (Canonicity)

If $\cdot \vdash M : \mathbf{bool}$*, then either* $M \equiv \mathbf{true}$ *or* $M \equiv \mathbf{false}$*.*

Proof.

We have $\mathbf{1} \xrightarrow{M} \mathbf{bool}$ in $\mathbb{C}$; by the universal property of the initial algebra $\mathbb{C}$, we have $\mathbf{1} \xrightarrow{!_{\mathbb{G}}(M)} \widetilde{\mathbf{bool}}$; considering the fibers of $\widetilde{\mathbf{bool}}$, we have $(\partial_1[\mathbf{N}] \circ !_{\mathbb{G}})(M) \in \{\mathbf{true}, \mathbf{false}\}$. But by initiality we have the following triangle in $\mathbf{Alg}[\mathbf{XTT}]$:

$$\begin{tikzcd} \mathbb{C} \arrow[r, "!_{\mathbb{G}}"] \arrow[dr, no head, "\mathbf{id}_{\mathbb{C}}"'] & \mathbb{G} \arrow[d, "\partial_1[\mathbf{N}]"] \\ & \mathbb{C} \end{tikzcd}$$

□

Outlook

XTT exhibits a strictness mismatch, making the combination of function comprehension and effective quotients unlikely; XTT ought to be viewed as a coherence construction which is developed *inside* a broader, univalent type theory which has the customary (higher) exactness properties.

We expect our metatheoretic techniques will scale to proving normalization of the infinite-dimensional cubical type theory (using a different nerve).

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