The agda-unimath library

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The agda-unimath library was founded on November 19th 2021 by Elisabeth Bonnevier, Jonathan Prieto-Cubides, and Egbert Rijke. Its first purpose is to formalize general mathematics from a univalent point of view, and its second purpose is to be an informative resource. Within a year, it became the largest and fastest growing library of formalized mathematics of any kind in the Agda proof assistant, including libraries for non-univalent mathematics. The library currently covers the following topics:

- Category theory (38 files)
- Commutative algebra (22 files)
- Elementary number theory (86 files)
- Finite group theory (18 files)
- Foundation (260 files)
- Graph theory (43 files)
- Group theory (112 files)
- Linear algebra (17 files)
- Order theory (43 files)
- Organic chemistry (8 files)

- Orthogonal factorization systems (8 files)
- Polytopes (1 file)
- Ring theory (30 files)
- Set theory (7 files)
- Structured types (36 files)
- Synthetic homotopy theory (29 files)
- Trees (24 files)
- Type theories (6 files)
- Univalent combinatorics (95 files)

Most of the work in the agda-unimath library has focused on introducing important concepts and building infrastructure for them. Some notable pieces of formalised mathematics in the agda-unimath library include: binomial types and Stirling types of the second kind, a characterization of the identity types of pushouts, concrete group theory, deloopings of groups via torsors, the delooping of the sign homomorphism, enriched directed and undirected graphs and trees, the equivalence of equivalence relations, partitions, surjective maps, and Σ-decompositions, the type of hydrocarbons, involutive types, the number of groups of order n (OEIS A000001), π-finite types, Russell's paradox for multisets, and symmetric identity types, symmetric H-spaces, and symmetric multivariable equivalences. The library contains most of the Introduction to Homotopy Type Theory textbook, and large portions of the Symmetry textbook, goes beyond the matrial of those texts in many ways, and contains significants amount of constructive algebra. In my talk I plan to give a global overview of the library, its philosphy, and discuss some of the novel mathematical ideas formalized in agda-unimath.

Foundational aspects of the agda-unimath library

The agda-unimath library is written in plain Agda, which implements a variant of Martin-Löf's dependent type theory. The options −−without−K, −−exact−split, −−no−import−sorts, and −−auto−inline are globally loaded. Furthermore, it postulates function extensionality,

univalence, truncations, type theoretic replacement, the interval, the circle, and homotopy pushouts.

Design philosophy of the agda-unimath library

The goal of the agda-unimath library to be an informative library of formalized univalent mathematics has direct implications to the design principles of the library. The files correspond to pages on the agda-unimath website, which have a layout similar to nlab pages, including informal sections explaining the idea behind a concept, its formal definition, and immediate properties. Each file in the agda-unimath project therefore focuses sharply on one concept or idea, or a single named theorem.

Users of a library of formalized should be able to find a concept without having to know how it has been formalized, what other concepts it depends on, or what more general concept it is an instance of. The folders in the agda-unimath library therefore represent broad subjects of mathematics, which may contain many files. Files names are concise descriptions of the concept or idea covered in them, and are listed alphabetically in the indexing file for the mathematical subject it belongs to. We avoid elaborate folder hierarchies which are common in other libraries of formalized mathematics, that often organize the content according to how it is formalized, for instance with folder names such as Data. We also avoid the library design to be dictated by the bootstrapping process that occurs at most foundational level. Our choice to take a "concepts-first" approach resulted in an organization of a library that is natural from a mathematical point of view.

The community of contributors to the agda-unimath library

28 people have contributed to the agda-unimath library, and we have a vibrant community with over 400 members on the Univalent Agda discord, which is shared with the communities of the 1lab, the cubical agda library, and the TypeTopology project.