In this talk I’ll introduce a new formulation of Turing reducibility based on the theory of higher modalities [RSS20] and building on some key ideas due to Hyland [Hyl82, Section 17].

Through cubical assemblies [SU21] we see that it is consistent with HoTT that all functions \( \mathbb{N} \to \mathbb{N} \) are computable. However, despite this, cubical assemblies do contain non computable functions: we can include the category of sets in cubical assemblies by first viewing a set as a uniform assembly, and then viewing the assembly as a discrete cubical assembly. We can access these functions from the internal language of cubical assemblies via double negation sheaves. Explicitly, using the results of [Swa22] we can define the modality, \( \nabla \), of 0-truncated \( \neg \neg \)-sheafification in cubical assemblies. We can then think of maps \( \chi : \mathbb{N} \to \nabla \mathbb{N} \) as external functions in sets that are not necessarily computable. Given \( \chi \) we can define an oracle modality \( \bigcirc \chi \) as the smallest modality forcing \( \chi \) to extend to a total function from \( \mathbb{N} \) to \( \mathbb{N} \), formally defined using nullification. Functions \( \mathbb{N} \to \mathbb{N} \) in the reflective subuniverse corresponding to \( \bigcirc \chi \) can be thought of as functions that are computable using \( \chi \) as an oracle.

The class of all modalities can be naturally viewed as a preorder by setting \( \bigcirc \chi \leq \bigcirc \chi' \) when every \( \bigcirc \chi' \)-modal type is \( \bigcirc \chi \)-modal. Applying this ordering to oracle modalities gives us a new way of looking at the preorder of Turing degrees, an important structure in computability theory.

I’ll discuss applications to the field of synthetic computability theory [Ric83, Bau06, Bau17, FJ23] and give some basic examples of oracle modalities interacting with higher types.

References


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