Deciding the cofibration logic of cartesian cubical type theories

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The core insight of Homotopy Type Theory is that the identity types of Martin-Löf type theory can naturally be interpreted as spaces of paths up to homotopy. Though ground-breaking, this insight left open crucial computational issues: for example, a general account of the operational meaning of paths between types. Cubical type theories are variants of HoTT which have since provided the missing operational meanings. Each includes as a central component a *cofibration logic*, which is used to specify and reason about select portions of paths of every dimension. The decidability of type checking presupposes the automation of the cofibration logic. In this talk, we study the decision problem of semantic entailment for the cofibration logics of several cubical type theories, and present a procedure which decides it. Notably, our procedure avoids the expense of normalizing cofibration formulas into equivalent disjunctive normal forms. We hence prove that entailment in the most expressive of these cofibration logics, which appears in [3], is complete for the second level of the polynomial hierarchy (specifically, for Π_2^{p}), and that entailment in each of the cofibration logics which appear in [2] and [1] is coNP-complete. These proofs include efficient reduction functions whose implementations will yield practical solvers which take advantage of modern SAT and 2QBF solving techniques.

References

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