Orbifolds in Synthetic Differential Cohesive HoTT*

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Informally, an orbifold is a smooth space whose points may have finitely many internal symmetries. Formally, however, the notion of orbifold has been presented in a number of different guises – from Satake's V-manifolds [4] to Moerdijk and Pronk's proper étale groupoids [2] – which do not on their face resemble the informal definition. The reason for this divergence between formalism and intuition is that the points of spaces cannot have internal symmetries in traditional, set-level foundations. The extra data of these symmetries must be carried around and accounted for throughout the theory. More drastically, maps between orbifolds presented in the usual ways cannot be defined pointwise.

In this talk — following the associated paper [3] — we will put forward a definition of orbifold in synthetic differential cohesive homotopy type theory: an orbifold is a microlinear type for which the type of identifications between any two points is properly finite. A set is properly finite when it is a discrete subquotient of a finite set. In homotopy type theory, a point of a type may have internal symmetries, and we will be able to construct examples of orbifolds by defining their type of points directly. Moreover, the mapping space between two orbifolds is merely the type of functions.

We will justify this synthetic definition by proving, internally, that every proper étale groupoid is an orbifold. In this way, we will show that the synthetic theory faithfully extends the usual theory of orbifolds. Along the way, we will investigate the microlinearity of higher types such as étale groupoids, showing that the methods of synthetic differential geometry generalize gracefully to higher analogues of smooth spaces. We will also investigate the relationship between the Dubuc-Penon [1] and open-cover definitions of compactness in synthetic differential geometry and show that any discrete, Dubuc-Penon compact subset of a second-countable manifold is subfinitely enumerable.

References

[1] Eduardo J. Dubuc and Jacques Penon. "Objets compacts dans les topos". In: *Journal of The Australian Mathematical Society* 40 (1986), pp. 203–217.

[2] Ieke Moerdijk and Dorette Pronk. "Orbifolds, sheaves and groupoids". In: *K-theory* 12 (1997), pp. 3–21.

[3] David Jaz Myers. *Orbifolds as microlinear types in synthetic differential cohesive homotopy type theory*. 2022. arXiv: 2205.15887 [math.AT].

[4] I Satake. "On a generalization of the notion of manifold". en. In: *Proc. Natl. Acad. Sci. U. S. A.* 42.6 (June 1956), pp. 359–363.

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