Computational Synthetic Homotopy Theory

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Computational variations of Homotopy Type Theory, like cubical type theories and H.O.T.T., provide new settings for developing synthetic homotopy theory. In these systems many things that hold only up to a path in Book HoTT hold definitionally, which leads to substantially simpler formal proofs of many results. It also opens up the possibility to prove results purely by normalization. A classic example is the Brunerie number–the cardinality of the fourth homotopy group of the 3-sphere. The automatic computation of this number was postulated by Brunerie in 2013 and we recently managed to simplify the definition of the number enough so that it becomes possible to compute it in just a few seconds in Cubical Agda. I will discuss this simplification and the computation in the talk, as well as other similar computer aided proofs in synthetic homotopy and cohomology theory that we have done. I will also discuss various numbers that we have not managed to get Cubical Agda to compute for us, hence providing a list of challenges for future computational proof assistants for HoTT.